CHAPTER 4
SEEPAGE PRINCIPLES

4-1. General Considerations. Seepage as used in this manual is defined as the flow of water through homogeneous saturated soil under steady-state conditions. Additionally, the soil particles, soil structure, and water are assumed incompressible and flow obeys Darcy's law. Thus transient conditions such as a wetting front or other movement of water in unsaturated soil, consolidation, and subsidence are not considered for analysis. Principles which characterize movement of energy through conducting media also apply to the movement of water through soils. Seepage has been modeled for study by using flow of electricity and heat. Both conditions are governed by Laplace's equation in homogeneous media. As explained in Chapter 2, water moves from a higher energy state to a lower energy state, and in seepage the difference in energy states is the amount of energy required to move the water through the soil, i.e., to overcome the soil's resistance to the flow of water. Chapter 4 will consider factors controlling seepage, equations describing seepage, methods of determining pressure distribution and pressures at particular points in the soil, and seepage quantities. For example, in figure 4-1(a): What is the uplift pressure at point 5? How much water will exit at point 8? How fast? Will the sand at point 8 be eroded? If the sheet pile at point 6 is removed, how will it affect pressure distribution beneath the dam?

4-2. Boundary Conditions.

a. Basis. The saturated soil which is considered for analysis must be defined by boundaries, permeability of the soil, and heads imposed upon the water. This section considers the types of boundaries which may define a particular porous soil mass considered for analysis. The nature and location of these boundaries are determined by a soils exploration program, assumptions based on engineering judgment and conditions imposed by the proposed design. Normally, simplifying assumptions are required in order to establish boundaries which will make analysis feasible. Generally, seepage analysis problems associated with dams will involve four possible types of boundaries (Harr 1962). Examples of the four general types of boundary conditions are shown in figure 4-1.

b. Impervious Boundaries. The interface between the saturated, pervious soil mass and adjacent materials such as a very low permeability soil or concrete is approximated as an impervious boundary. It is assumed that no flow takes place across this interface, thus flow in the pervious soil next to the impervious boundary is parallel to that boundary. In figure 4-1, lines AB and 1-8 are impervious boundaries.

c. Entrances and Exits. The lines defining the area where water enters or leaves the pervious soil mass are known as entrances or exits, respectively. Along these lines (O-l and 8-G in figure 4-1(a) and AD and BE in figure 4-1(b)) are lines of equal potential; that is, the piezometric level is the same all along the line regardless of its orientation or shape. Flow is perpendicular to an entrance or exit. Entrances and exits are also called reservoir boundaries (Harr 1962).

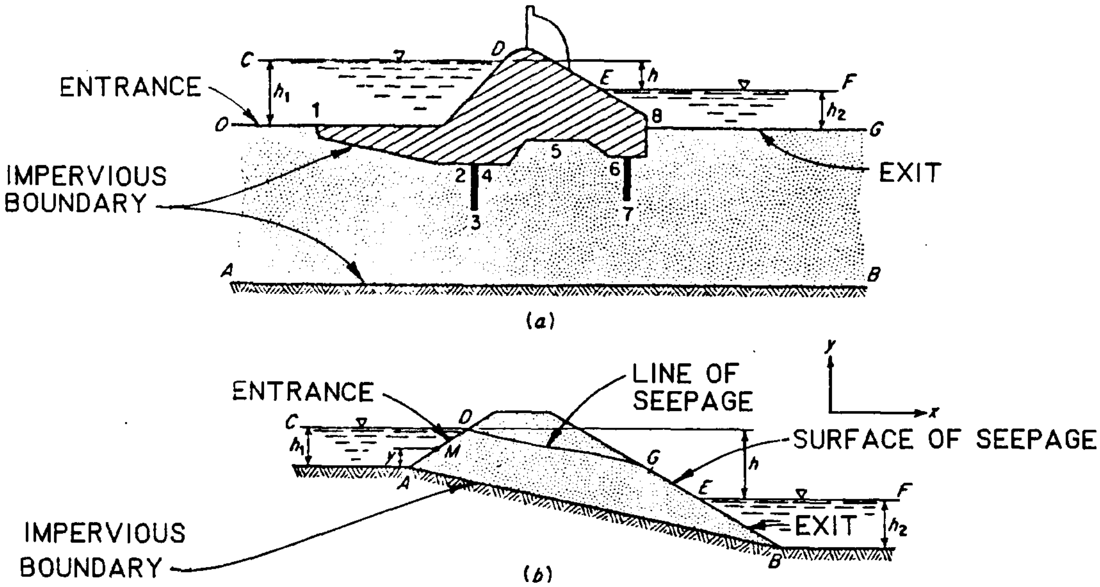


Figure 4-1. Examples of boundary conditions (courtesy of McGraw-Hill Book Company[180])

d. Surface of Seepage. The saturated pervious soil mass may have a boundary exposed to the atmosphere and allow water to escape along this boundary, line GE, figure 4-1(b). Pressure along this surface is atmospheric. The surface of seepage may also be called a seepage face.

e. Line of Seepage. Known also as the free surface, this boundary is located within the pervious soil where water is at atmospheric pressure, line DG, figure 4-1(b). Because of capillary forces, the saturated zone of pervious soil extends slightly above the line of seepage, but this capillary zone rarely has significant influence on seepage analysis. Whereas the first two boundaries are normally defined by the geometric boundaries of the saturated porous soil mass, the line of seepage is not known until the flow distribution within the pervious soil is known. Again, as for an impervious boundary, the assumption is made that no flow takes place across the line of seepage, thus flow in the pervious soil next to this boundary is parallel to the boundary.

4-3. Confined and Unconfined Flow Problems. Two general cases of seepage are considered in this manual: confined and unconfined flow. Confined flow exists in a saturated pervious soil mass which does not have a line of seepage boundary. Figure 4-1(a) is an example of confined flow. Unconfined flow, figure 4-1(b), exists when the pervious soil mass has a line of seepage. Thus confined flow has all boundaries defined while for unconfined flow the surface of seepage and line of seepage must be defined in the analysis.

4-4. Laplace's Equation.

a. Seepage Analysis. In order to do a seepage analysis, a general model describing the phenomena of seepage must be available. Supplied with specific boundary conditions and soil properties, this model can be used to determine head and flow distribution and seepage quantities. The Laplace equation is the mathematical basis for several models or methods used in seepage analysis.

b. Basis of Laplace's Equation. Figure 4-2 shows a general seepage condition from which an element is taken. Development of Laplace's equation depends on six assumptions:

(1) Heads h_1 and h_2 are constant and thus flow is steady state.

(2) Water is incompressible.

(3) Volume of voids does not change--soil is incompressible.

(4) Flow is laminar--Darcy's law applies.

(5) The element has a dimension, dy , into the plane of the figure which gives an element volume but no flow takes place perpendicular to the plane of the figure, i.e., the flow is two-dimensional.

(6) The saturated pervious soil stratum is homogeneous. From figure 4-2(b) let:

v_x, v_z = components of discharge velocity in x and z directions, respectively

$i_x = -\frac{\partial h}{\partial x}$ hydraulic gradient in the x direction

$i_z = -\frac{\partial h}{\partial z}$ hydraulic gradient in the z direction

(The minus sign indicates that gradient is in a direction opposite increasing piezometric head.) Assumptions 1, 2, and 3 assure continuity of flow which means that water entering the element per unit of time, q_e (where $q_e = v_x$ dz dy + v_z dx dy) equals water leaving the element per unit of time, q_ℓ (where $q_\ell = v_x\, dz\, dy + \frac{\partial v_x}{\partial x} + dx\, dz\, dy + v_z\, dx\, dy + \frac{\partial v_z}{\partial z}\, dz\, dx\, dy$). Setting q_e equal to q_ℓ gives:

$$\frac{\partial v_x}{\partial x} dx\ dz\ dy + \frac{\partial v_z}{\partial z} dz\ dx\ dy = 0$$

or

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z} = 0 \qquad (4\text{-}1)$$

Using Darcy's law, $v = ki$ and assuming the same permeability in the x and z directions:

$$v_x = ki_x = -k \frac{\partial h}{\partial x}$$

$$v_x = k\ i_z = -k \frac{\partial h}{\partial z}$$

kh is called a potential or velocity potential and is normally given the symbol Φ . Thus

$$\Phi = kh$$

and

$$v_x = -\frac{\partial \Phi}{\partial x} \quad v_z = -\frac{\partial \Phi}{\partial z}$$

Substituting into equation 4-1 gives

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial z^2} = 0 \qquad (4\text{-}2)$$

which is a form of the Laplace equation for laminar, two-dimensional flow in homogeneous, isotropic, porous media. The development here follows Terzaghi 1943. Rigorous developments can be found in Bear 1972, Cedergren 1977, and Harr 1962.

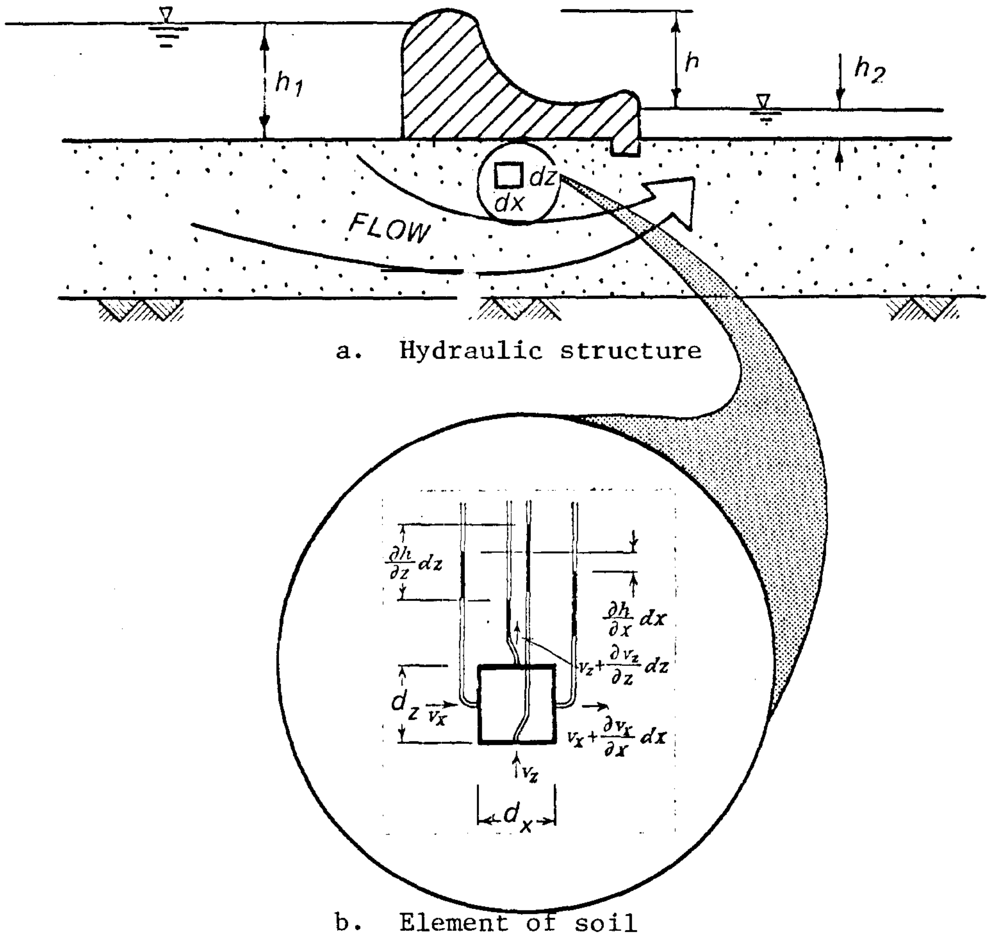


Figure 4-2. Flow of water through saturated pervious soil beneath a hydraulic structure (courtesy of John Wiley and Sons[274])

4-5. Methods for Solution of Laplace's Equation. Solutions to steady-state, laminar flow, seepage problems must solve Laplace's equation. Several methods have been developed to solve exactly or approximately Laplace's equation for various cases of seepage, figure 4-3 (Radhakrishnan 1978). One of the most widely used methods, the flow net, can be adapted to many of the underseepage and through-seepage problems found in dams and other projects involving hydraulic structures. This method will be covered in detail in Section 4.6.

a. Models. Models which scale or simulate the flow of water in porous media can provide a good feel for what is occurring during seepage and allow a physical feel for the reaction of the flow system to changes in head, design geometry, and other assumptions. Appendix B contains examples of the various model types.

(1) As previously mentioned, processes which involve movement of energy due to differences in energy potential operate by the same principles as

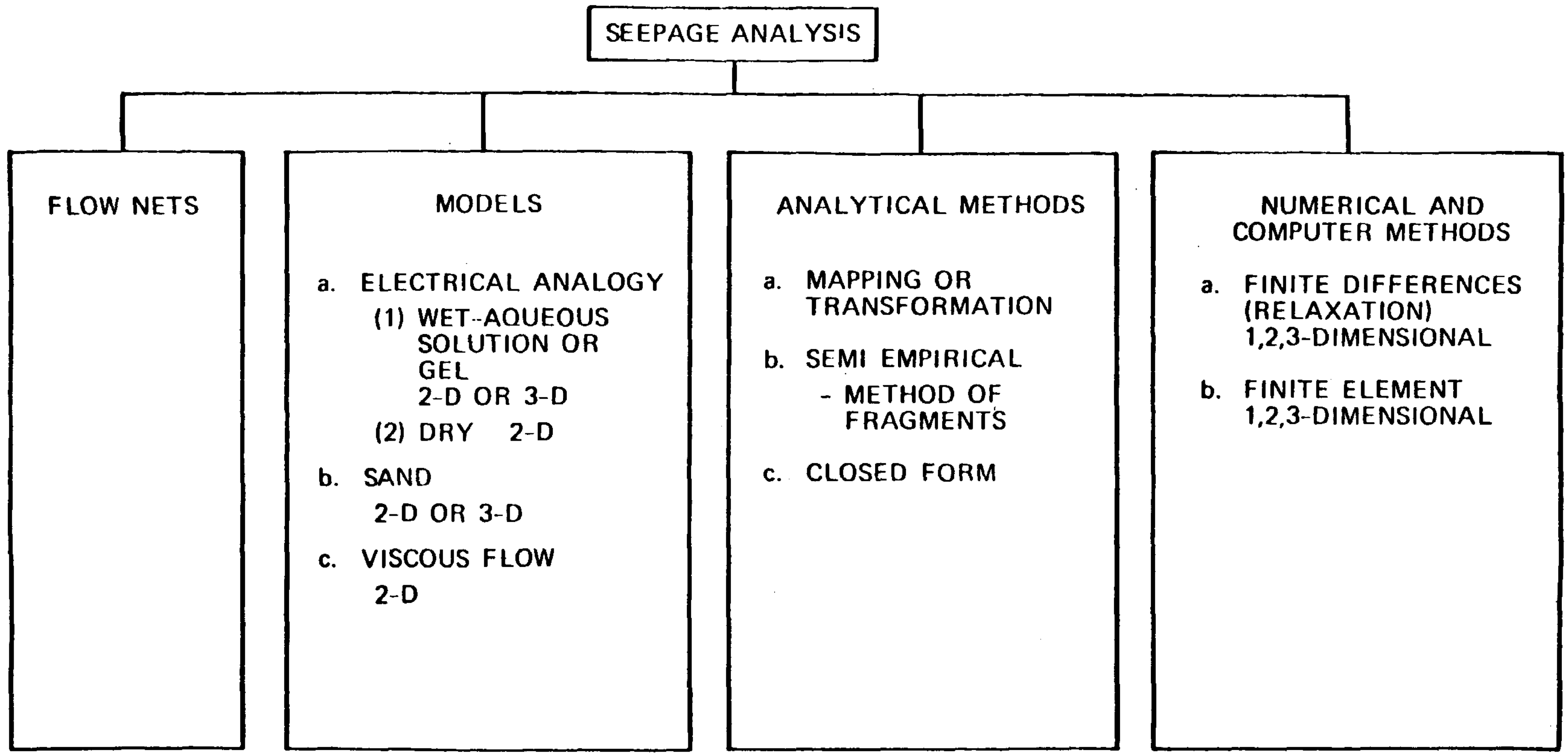


Figure 4-3. Seepage analysis methods (from Radhakrishnan[66])

movement of confined ground water. These processes include electricity and heat flow which have been used as seepage analogies. Electrical analogies have proven particularly useful in the study of three-dimensional problems and in problems where geometric complexities do not allow adequate simplifying assumptions for analytical methods. "Wet" electrical analogies normally use a conducting aqueous solution or gel to model the volume of the confined, saturated, porous soil. Wet models are well suited to projects where an irregular structure penetrates a confined aquifer. By probing the gel or solution when a set potential or voltage is applied across it, electrical potential can be determined at various points of interest in the model aquifer (McAnear and Trahan 1972, Banks 1963, 1965). When field conditions can be characterized by a two-dimensional plan or section, conducting paper models may be used to inexpensively determine the effect of various configurations on the flow and pressures in the aquifer, figure 4-4 (Todd 1980).

(2) Sand models which may use prototype materials can provide information about flow paths and head at particular points in the aquifer. The sand or porous material may be placed underwater to provide a homogeneous condition, or layers of different sand sizes may be used to study effects of internal boundaries or layers. If the flow is unconfined and the same material is used for model and prototype, the capillary rise will not be scaled and must be compensated for in the model. Flow can be traced by dye injection and heads determined by small piezometers. Disadvantages include effects of layering when the porous material is placed, difficulty in modeling prototype permeability and boundary effects. Prickett (1975) provides examples of sand tank models and discusses applications, advantages, and disadvantages.

(3) Viscous flow models have been used to study transient flow (e.g., sudden drawdown) and effects of drains. This method depends on the flow of a viscous fluid such as oil or glycerin between two parallel plates and is normally used to study two-dimensional flow. As with sand models, dye can be used to trace flow lines.' Construction is normally complicated and operation requires care since temperature and capillary forces affect the flow. Flow must be laminar, which can be difficult to achieve at the boundaries or at sharp changes in boundary geometry.

b. Analytical Methods.

(1) Harr (1962) explains the use of transformations and mapping to transfer the geometry of a seepage problem from one complex plane to another. In this manner, the geometry of a problem may be taken from a plane where the solution is unknown to a plane where the solution is known. While this method has been used to obtain solutions to general problems it is not frequently used for solutions to site-specific seepage problems since it requires the use of complex variable theory and proper choice of transformation functions.

(2) Pavlovsky (1936, 1956) developed an approximate method which allows the piecing together of flow net fragments to develop a flow net for the total seepage problem. This method, termed the Method of Fragments, allows rather complicated seepage problems to be resolved by breaking them into parts, analyzing flow patterns for each, and reassembling the parts to provide an

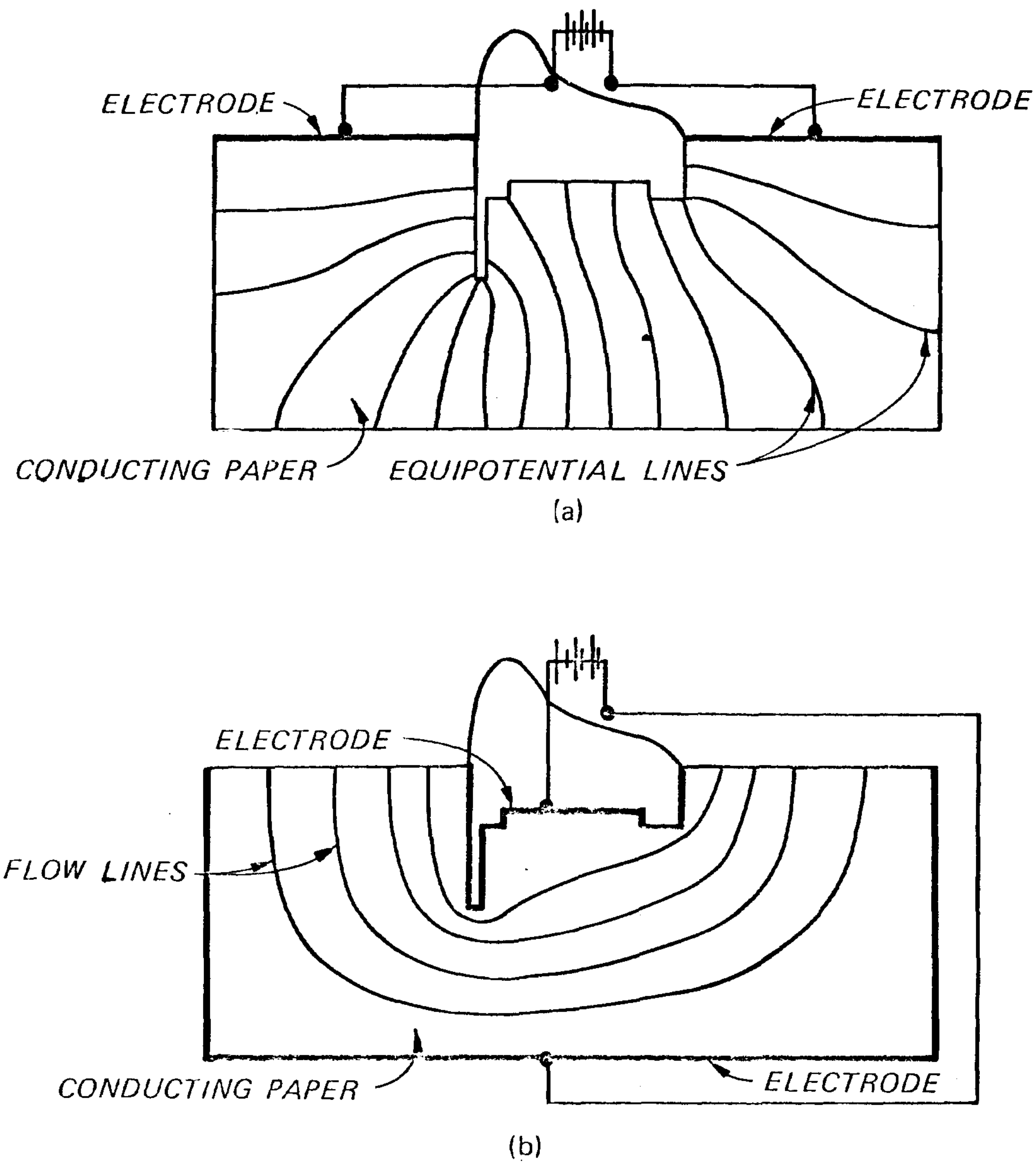


Figure 4-4. Use of two-dimensional conducting paper to find flow lines and equipotential lines (courtesy of John Wiley and Sons[279])

overall solution. Appendix B contains details of the Methods of Fragments based on Harr's (1962) explanation of Pavlovsky's work.

(3) Closed form solutions exist for simpler seepage conditions such as flow to a fully penetrating well with a radial source (Muskat 1946). Seepage problems associated with dams typically require approximate solutions because of complicated flow conditions.

c. Numerical and Computer Methods. Computer models are used to make acceptable approximations for the Laplace equation in complex flow conditions. The two primary methods of numerical solution are finite difference and finite element. Both can be used in one-, two-, or three-dimensional modeling. Several computer programs for these methods are available within the Corps of Engineers (Edris and Vanadit-Ellis 1982).

(1) The finite difference method solves the Laplace equations by approximating them with a set of linear algebraic equations. The flow region is divided into a discrete rectangular grid with nodal points which are assigned values of head (known head values along fixed head boundaries or points, estimated heads for nodal points that do not have initially known head values). Using Darcy's law and the assumption that the head at a given node is the average of the surrounding nodes, a set of N linear algebraic equations with N unknown values of head are developed (N equals number of nodes). Simple grids with few nodes can be solved by hand. Normally, N is large and relaxation methods involving iterations and the use of a computer must be applied. Appendix B provides details of this method.

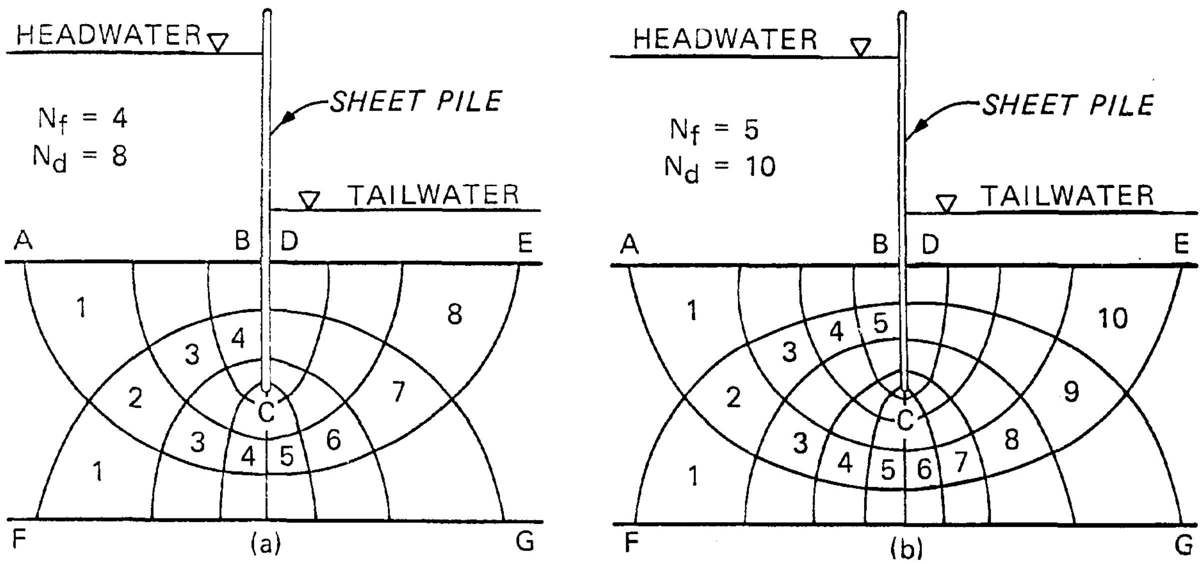
(2) The finite element method is a second way of numerical solution. This method is also based on grid pattern (not necessarily rectangular) which divides the flow region into discrete elements and provides N equations with N unknowns. Material properties, such as permeability, are specified for each element and boundary conditions (heads and flow rates) are set. A system of equations is solved to compute heads at nodes and flows in the elements. The finite element has several advantages over the finite difference method for more complex seepage problems. These include (Radhakrishnan 1978):

(a) Complex geometry including sloping layers of material can be easily accommodated.

(b) By varying the size of elements, zones where seepage gradients or velocity are high can be accurately modeled.

(c) Pockets of material in a layer can be modeled.

4-6. Graphical Method for Flow Net Construction. Flow nets are one of the most useful and accepted methods for solution of Laplace's equation (Casagrande 1937). If boundary conditions and geometry of a flow region are known and can be displayed two dimensionally, a flow net can provide a strong visual sense of what is happening (pressures and flow quantities) in the flow region. Equation 4-2, paragraph 4.4, is an elliptical partial differential equation whose solution can be represented by sets of orthogonal (intersecting at right angles) curves. One set of curves represents flow paths of water through the porous media while curves at right angles to the flow paths show the location of points within the porous media that have the same piezometric head. The former are called flow lines, the latter equipotential lines. The flow net is a singular solution to a specific seepage condition, i.e., there is only one family of curves that will solve the given geometry and boundary conditions. This does not mean that a given problem will have only one flow net--we may choose from the family of curves different sets of curves to define the problem, figure 4-5. The relationship between the number of equipotential drops, N_d , and flow channels, N_f , does not change. A brief study of figure 4-5 will provide a feel for where quantity of flow is greatest, velocity highest, and gradient highest, i.e., in the area of the porous soil nearest the sheet pile (flow channel 4, figure 4-5(a); flow channel 5, figure 4-5(b)). This section draws upon several publications which give



a. Net drawn for four flow channels.

b. Net drawn for five flow channels.

Figure 4-5. Flow net for a sheetpile wall in a permeable foundation (from U. S. Army Engineer District, Little Rock[92])

detailed explanation of flow net derivation and drawing instructions (Casagrande 1937; Cooley, Harsh, and Lewis 1972; Soil Conservation Service 1973; and Cedergren 1977). One of the best ways to develop an understanding of seepage and flow nets is to study well-drawn flow nets found in these and other references and to practice drawing them.

a. Assumptions for Flow Net Construction. In order to draw a flow net, several basic properties of the seepage problem must be known or assumed:

(1) The geometry of the porous media must be known.

(2) The boundary conditions must be determined (see paragraph 4.2).

(3) The assumptions required to develop Laplace's equation must hold (see paragraph 4.4b).

(4) The porous media must be homogeneous and isotropic (anisotropic conditions are dealt with in paragraph 4.7).

b. Guidelines for Flow Net Drawing. Once the section of porous media and boundary conditions are determined, the flow net can be drawn following general guidelines:

(1) Determine flow conditions at the boundaries:

(a) Flow will be along and parallel to impermeable boundaries lines BCD and FG, figure 4-5.

(b) Entrances and exits are equipotential lines, lines AB and DE, figure 4-5, with flow perpendicular to them.

(c) Flow will be along and parallel to a line of seepage--line AB, figure 4-6.

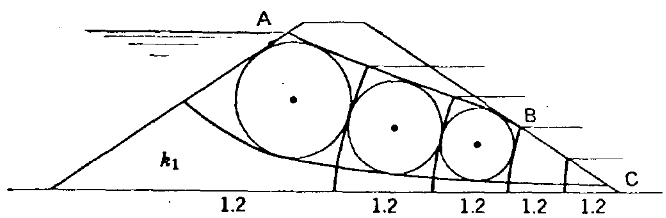


Figure 4-6. Seepage through an embankment underlain by an impermeable foundation (courtesy of John Wiley and Sons[155])

(d) Entrance and exit conditions for a line of seepage are shown in figure 4-7 under "Conditions for Point of Discharge."

This will provide a feel for the flow net.

(2) Equipotential and flow lines must meet at right angles and make curvilinear squares. Usually, it is best to make either the number of flow channels a whole number (if the number of flow channels is a whole number, the number of equipotential drops will likely be fractional).

(3) Generally, a crude flow net should first be completed and adjustments applied throughout the net rather than defining one portion since refinement of a small portion tends to shift the whole net.

(4) The initial emphasis should be on getting intersections of flow lines and equipotential lines at 90°, then shifting lines to form squares.

(5) If, in the finished flow net, either equipotential drops or flow channels end up as a whole number plus a fractional line of squares (equipotential drop or flow channel), this should not be a problem but must be used in any calculations based on the flow net. It is convenient to locate a partial equipotential drop in an area of uniform squares since this will make accurate estimation of the fraction easier.

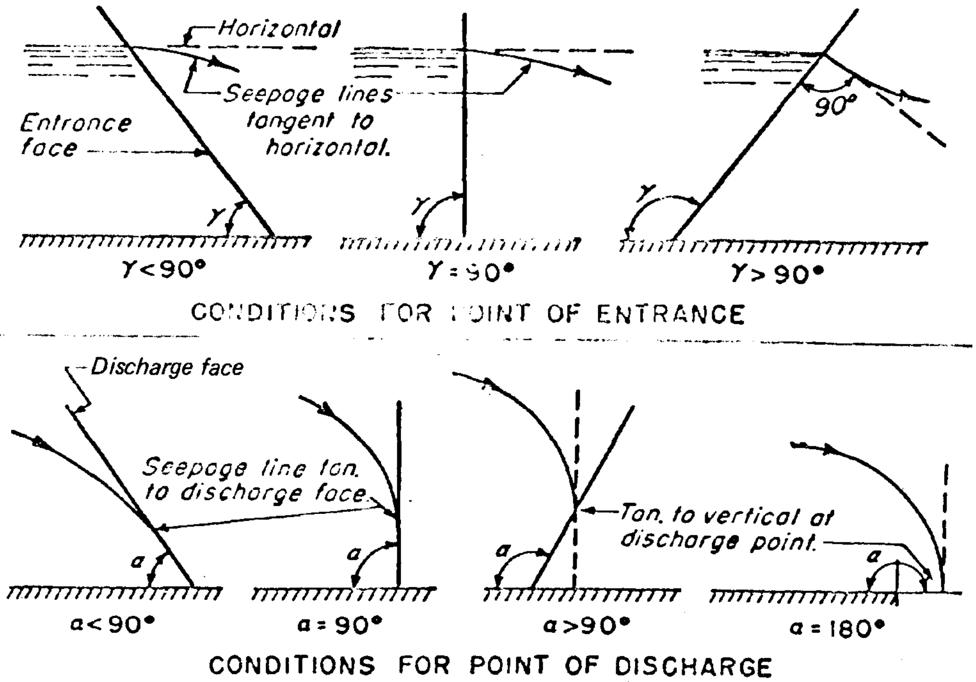


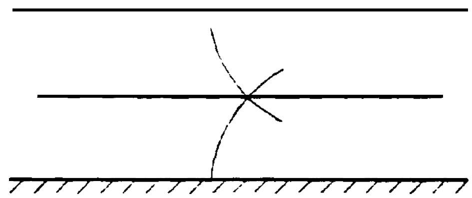
Figure 4-7. Entrance and discharge (exit) conditions for a line of seepage (courtesy of New England Waterworks Association[151])

(6) Use only enough flow lines and equipotential lines to bring out flow net definition. If more information is needed in particular areas, the squares may be subdivided into smaller squares for more detail of flow and pressure distribution.

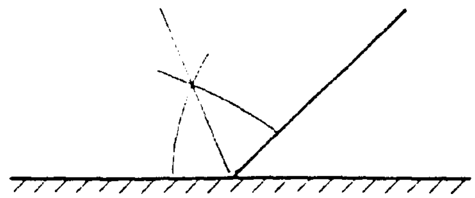
(7) As shown in figure 4-6, equipotential line intersections at a line of seepage, line AB, and a surface of seepage or discharge face, line BC, are controlled by elevation since pressure is atmospheric along these lines. Along the discharge face BC, the equipotential lines and flow lines do not form squares since the discharge face is not a flow line or an equipotential line but a line at atmospheric pressure and changing elevation potential.

(8) Figures 4-7 and 4-8 provide some guidelines for entrances and exits and particular areas within the flow region.

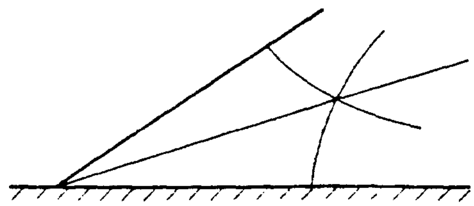
For foundations, furthermost upstream and downstream flow lines and equipotential lines should intersect at or near the center of the pervious foundation.



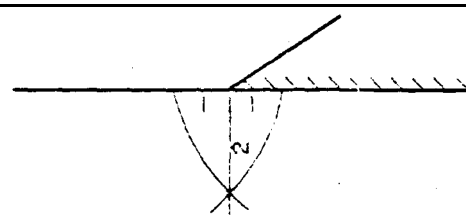
The flow line and equipotantial line nearest an angle should intersect on the bisector of the angle.



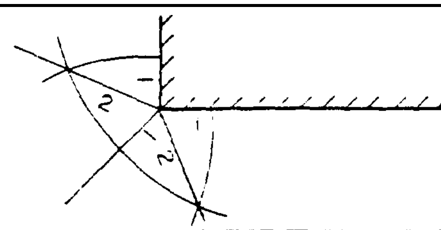
Same as (b) except for an upstream toe on an impervious foundation.



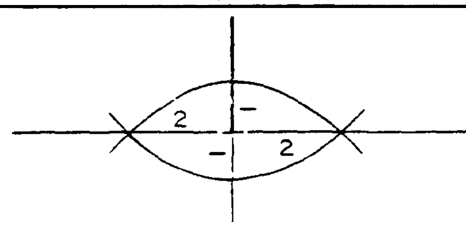
2:1 length ratios to establish shape of the “square” in a pervious foundation at the toe of an impervious fill.



2:1 length ratios used with angle bisectors to shape flow around an imbedded 90-degree angle.



2:1 length ratios to establish flow directions beneath a thin cutoff wall taken to the midpoint of the pervious stratum.



Subdivide to check odd-shaped "squares”. Resulting smaller odd-shaped "squares” should have the general shape of the one subdivided.

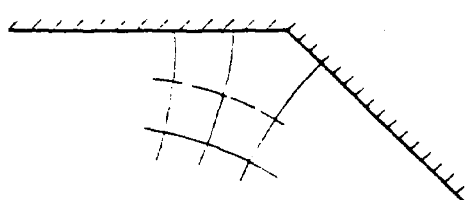


Figure 4-8. Guides for flow net construction (from U. S. Department of Agriculture[123])

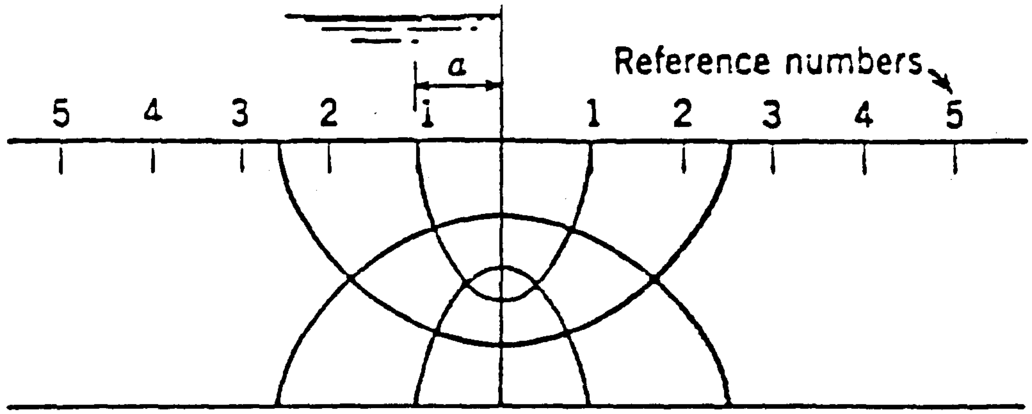
(9) It is helpful to lay out the boundaries which will contain the flow net in ink and use a soft pencil and eraser to develop the flow net to final form.

(10) Accuracy of squares may be checked by drawing diagonals for a square or subdividing the square by sketching an additional flow line and equipotential line orthogonal to it (ad infinitum). The diagonals should be smooth curves intersecting at right angles. Also, if the intersection of two flow lines and two equipotential lines is a square, a circle, tangent to each of the sides, may be inscribed within the square.

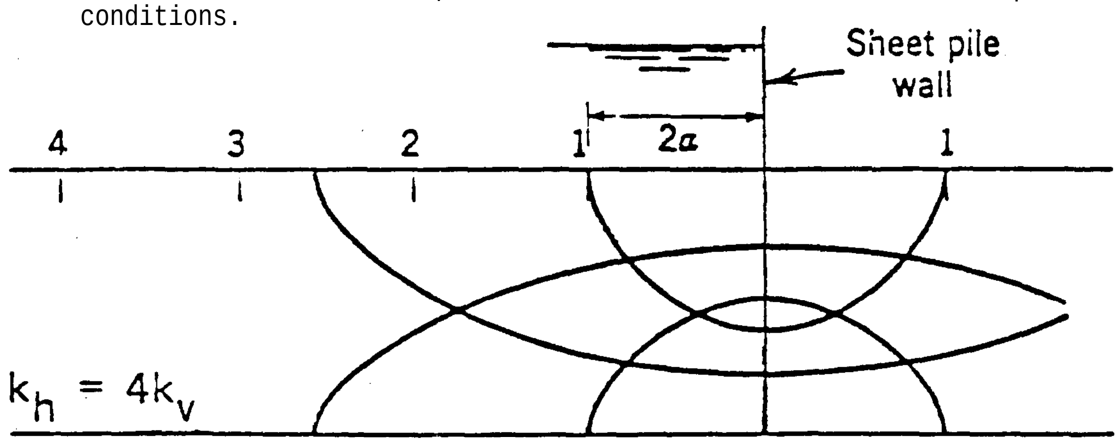
(11) For calculation of seepage quantity only a crude flow net is required. Accurate flow nets are required to determine pressure distribution.

4-7. Flow Net for Anisotropic Soil. Most naturally occurring soils and many man-placed soils have greater horizontal permeabilities than vertical. This affects the shape of a flow net since the flow net provides a solution to Laplace's equation which is based on the assumption of an isotropic porous media (paragraph 4.4b). To compensate for anisotropy, the dimensions of the porous media are changed by the square root of the ratio of the two permeabilities. If k_h is the horizontal permeability and k_v is the vertical permeability, then the horizontal dimensions of the porous media cross section are changed by a ratio of $\sqrt{k_v/k_h}$, e.g., if the base of a dam is 300 feet, then it would be changed by a factor of $\sqrt{k_v/k_h}$, or would be 300 feet times $\sqrt{k_v/k_h}$. The same ratio would be applied to all other horizontal dimensions to produce a transformed section. Next the flow net is drawn on the transformed section, as described in paragraph 4-6. Then the section, including the flow net, is returned to the original (true section) which produces a nonsquare flow net. Computations are made using the nonsquare flow net just as a square flow net is used for isotropic conditions. This procedure is illustrated in figures 4-9 and 4-10. In the same manner, dimensions in the vertical direction could be changed by the factor $\sqrt{k_h/k_v}$, square or normal flow net drawn on the transformed section, then returned to true section. Pore pressure distribution and hydrostatic uplift may be taken from either section while gradient and magnitude of seepage forces must be determined from the true section.

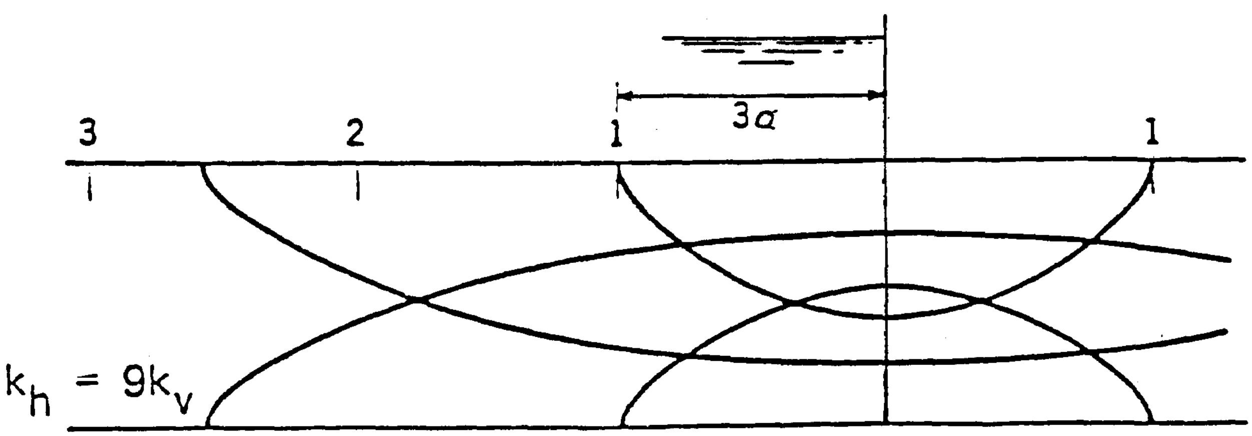
4-8. Flow Net for Composite Sections. Commonly, projects requiring seepage analysis involve different soils with different permeabilities, e.g., stratified foundation materials and zoned dams. Certain rules apply to flow lines, equipotential lines, and lines of seepage crossing internal boundaries between soils of different permeabilities. Figure 4-11 illustrates the deflection of flow lines and equipotential lines at interfaces. The essential principle is that the more permeable soil allows the same amount of water to flow with less restriction, thus drops in potential within the higher permeability soil will be farther apart (i.e., less energy loss in the higher permeability soil for the same length of flow as in the low permeability soil). It should be noted in figure 4-11 that when flow goes from lower permeability soil to higher permeability soil, the distance between flow lines decreases (flow channel gets smaller) and the distance between equipotential drops increases, Figures 4-12 through 4-14 are examples of flow net construction for seepage through soils of



a. True section for $k_h = k_v$ and transformed section for anisotropic conditions.



b. True section for flow net in a for $k_h = 4k_v$



c. True section for flow net in a for $k_h = 9k_v$

Figure 4-9. Flow nets constructed on transformed section and redrawn on true section (courtesy of John Wiley and Sons[155])

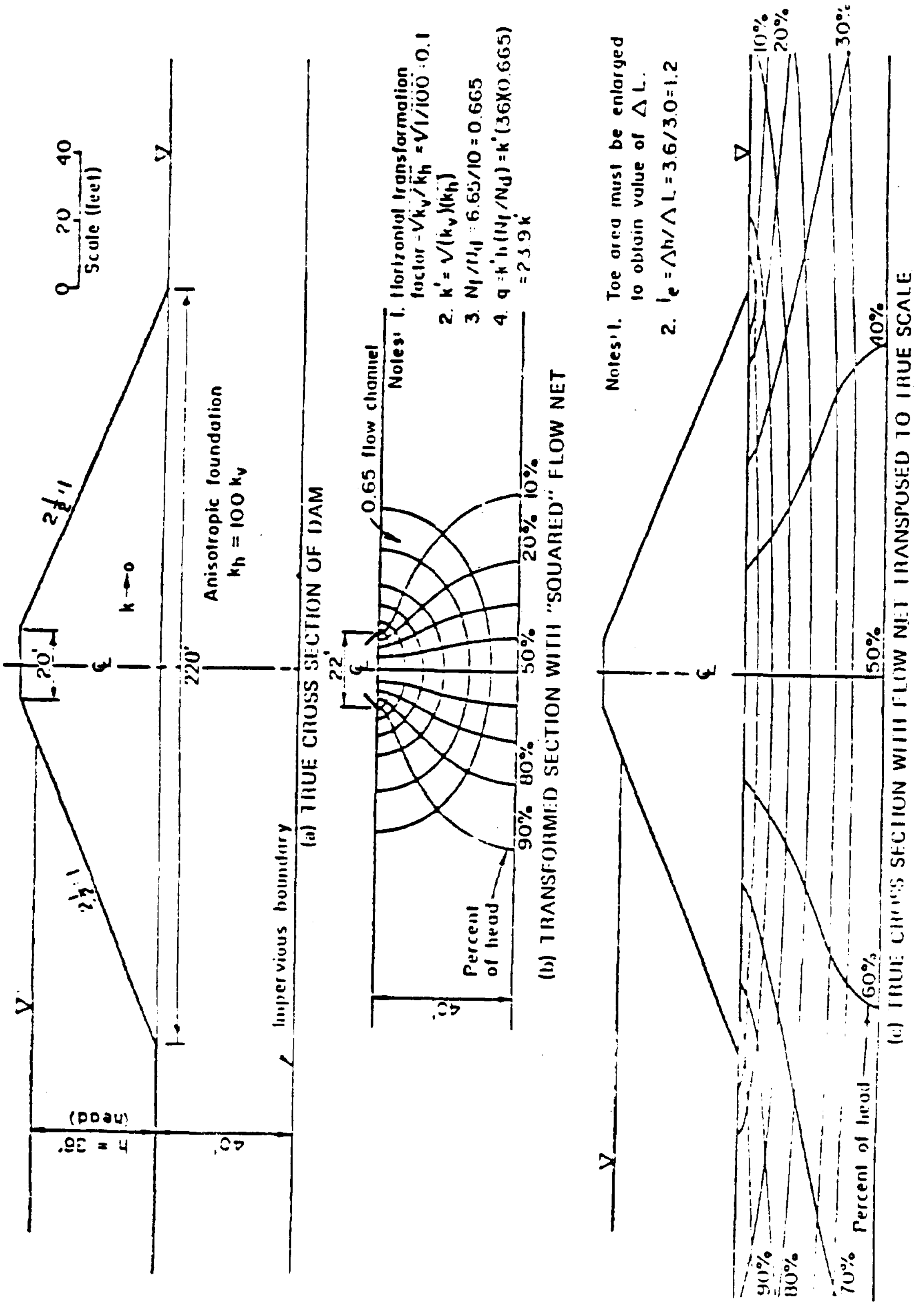


Figure 4-10. Example of technique for developing a flow net for an anisotropic foundation (from U. S. Department of Agriculture[123])

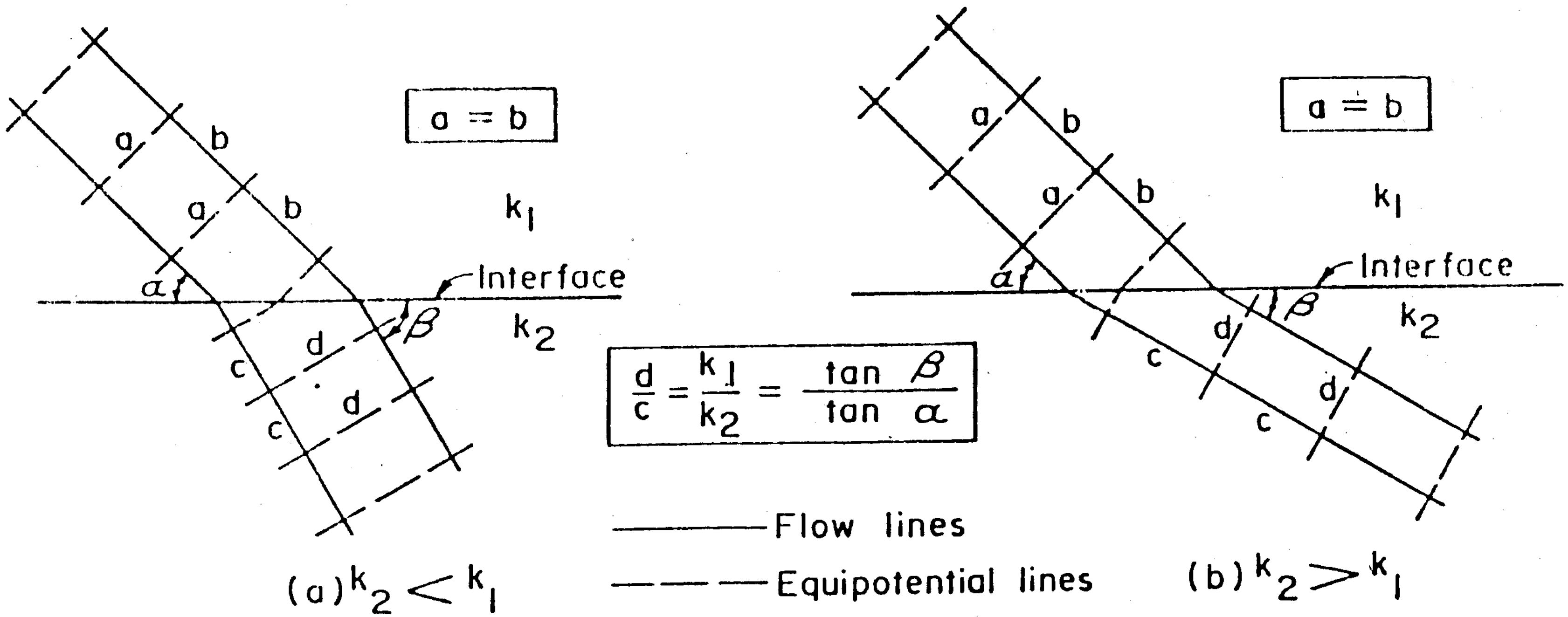
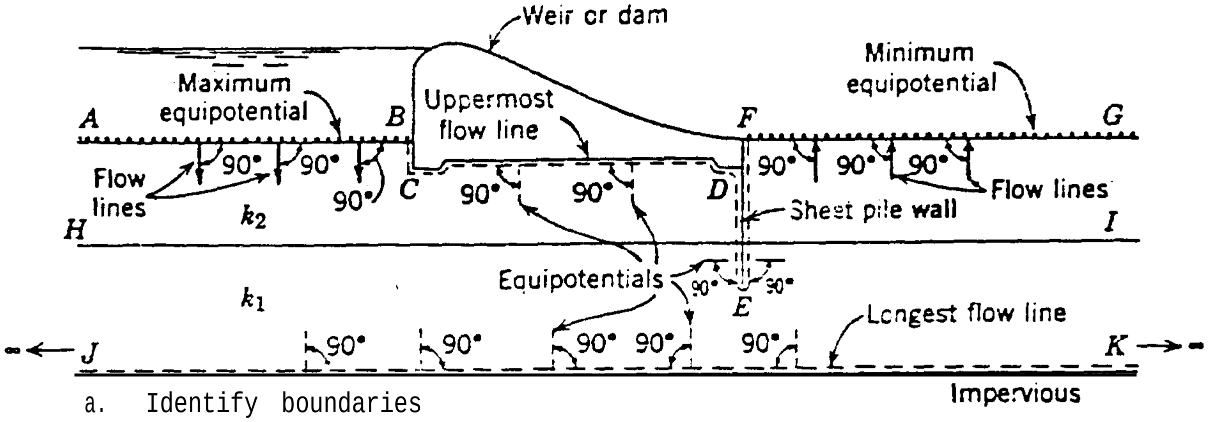
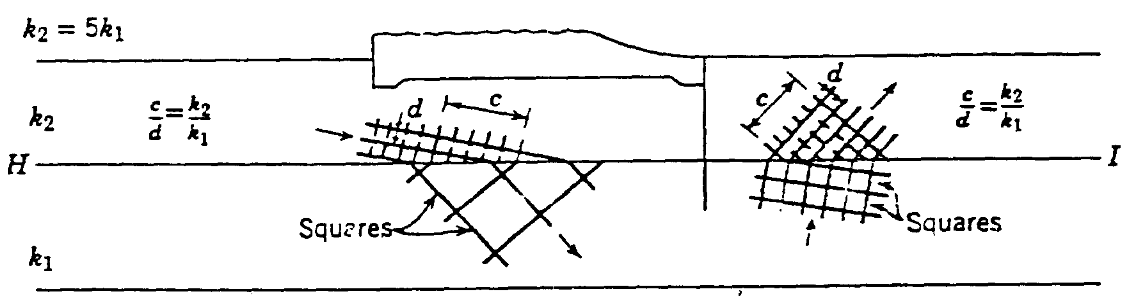


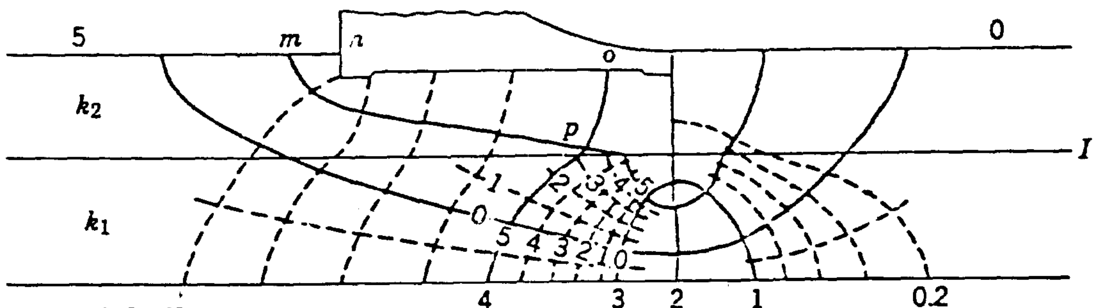
Figure 4-11. Deflections of flow lines at interfaces of soils having different permeabilities (courtesy of New England Waterworks Association[151])



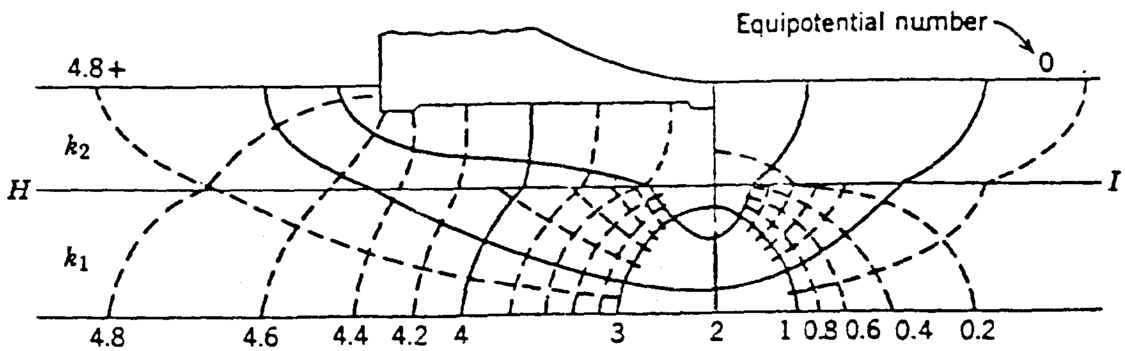
a. Identify boundaries



b. Determine deflection of flow net at interface between soils with different permeabilities.

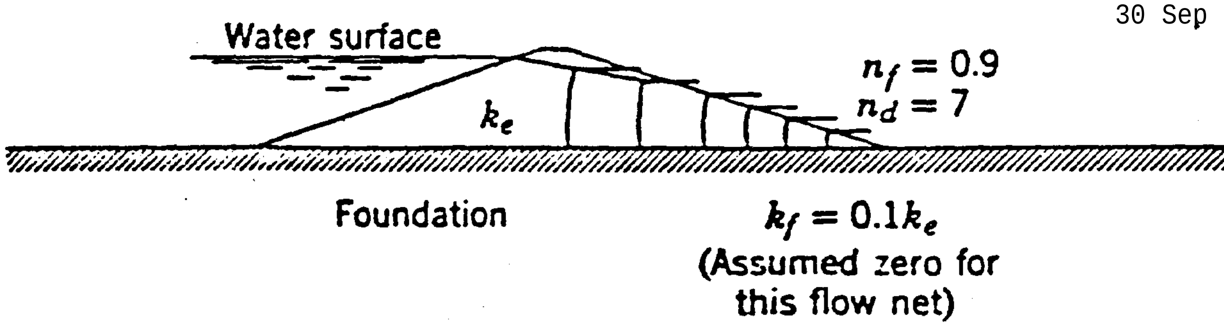


c. Trial flow net

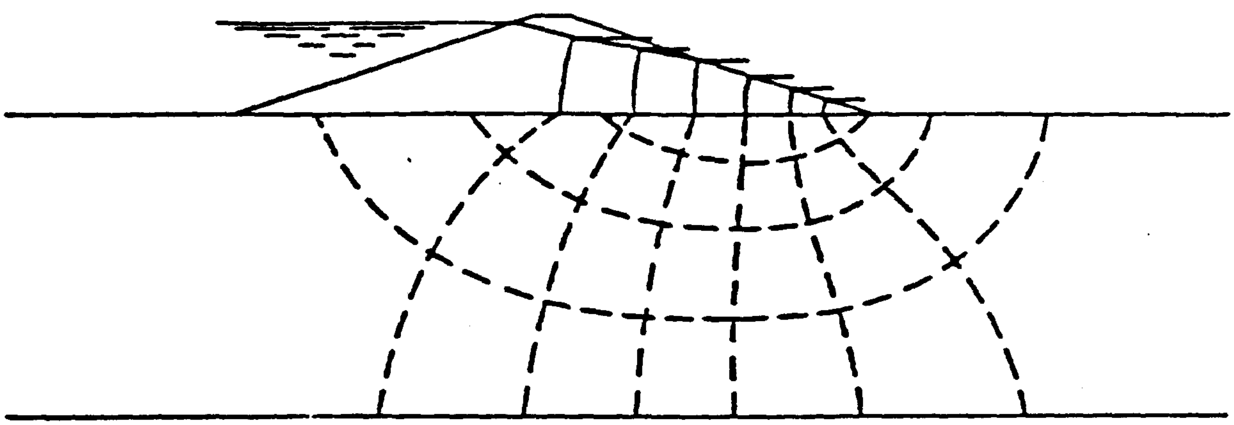


d. Final flow net

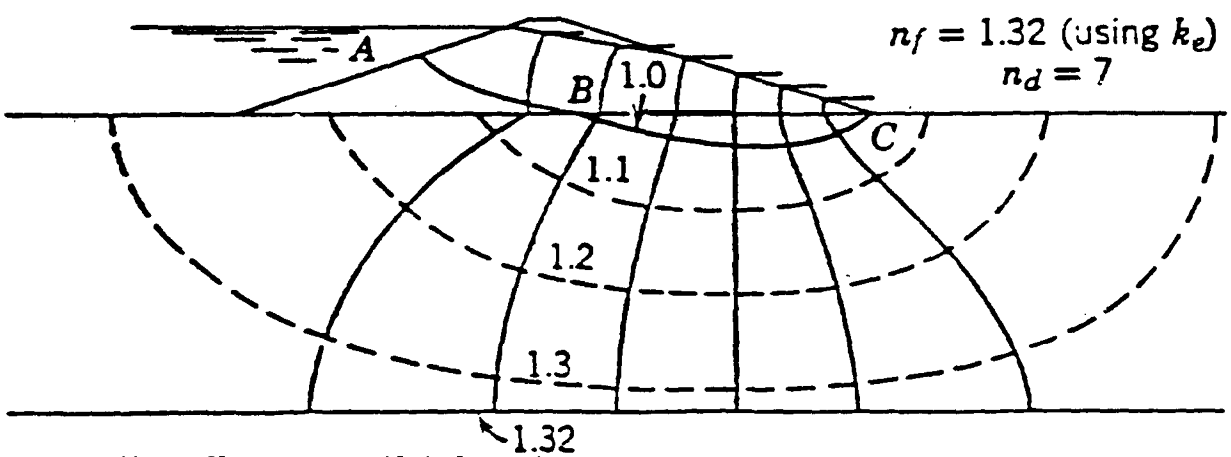
Figure 4-12. Flow net construction for a composite section (courtesy of John Wiley and Sons[155])



a. Construct flow net assuming an impermeable foundation.

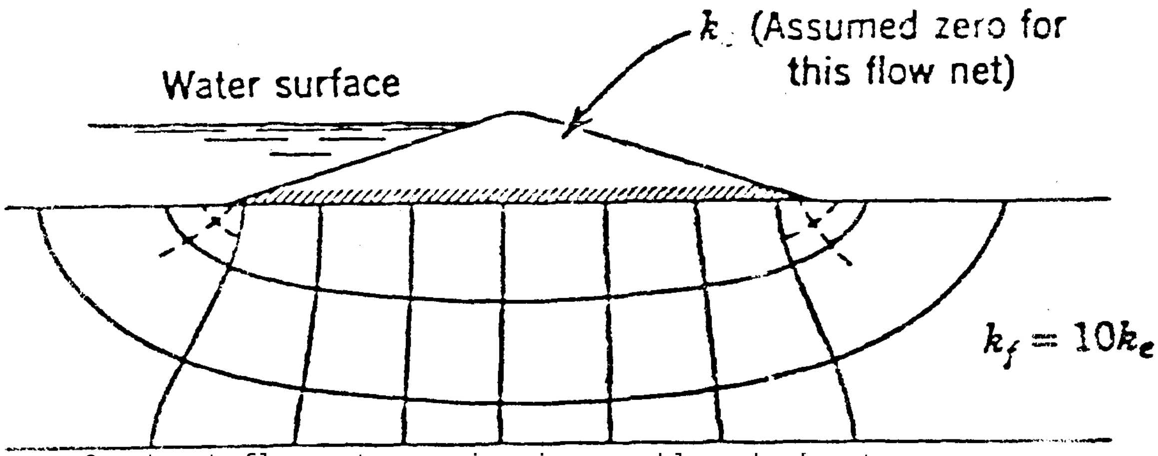


b. External equipotential lines into foundation without adjusting lines of net in dam.

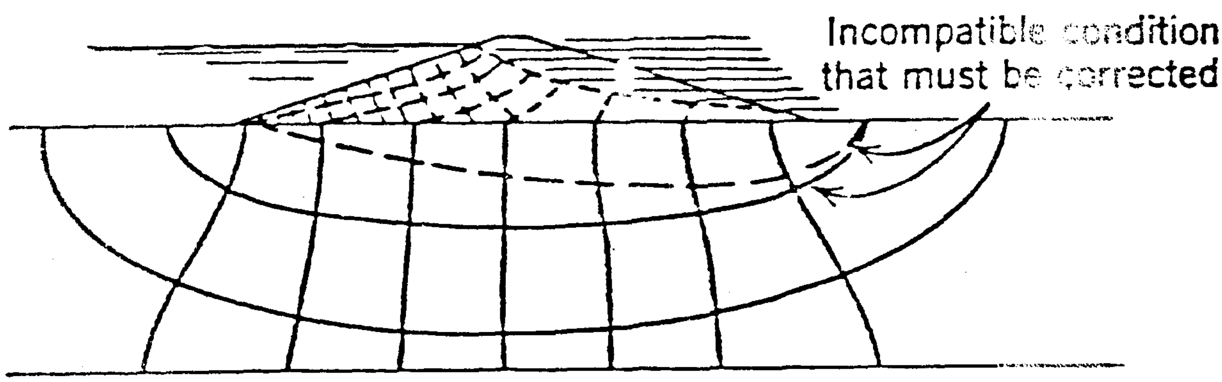


c. Adjust flow net until balanced.

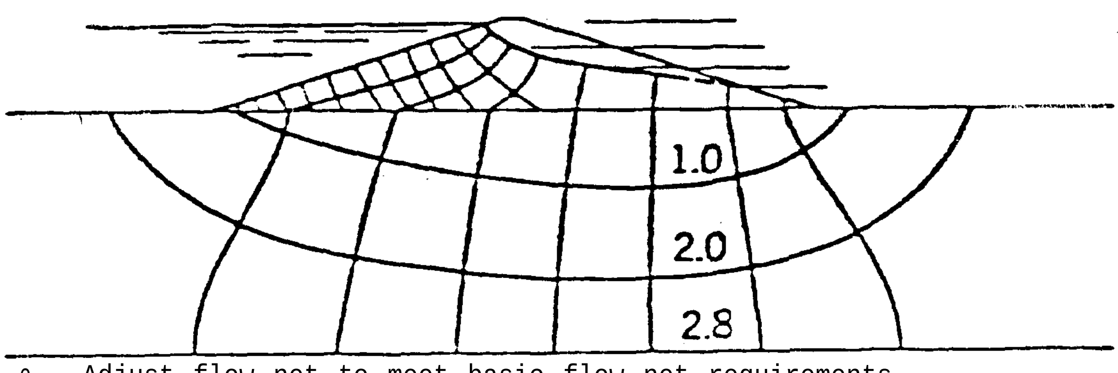
Figure 4-13. Flow net construction for embankment on a foundation of lower permeability (courtesy of John Wiley and Sons[155])



a. Construct flow net assuming impermeable embankment



b. Extend equipotentials up into dam locating initial position of line of seepage



c. Adjust flow net to meet basic flow net requirements.

Figure 4-14. Flow net construction for an embankment on a foundation of higher permeability (courtesy of John Wiley and Sons[155]) differing permeabilities. In all cases, flow lines and equipotential lines maintain continuity across the interface between the soils though direction will change abruptly. Additionally, the number of flow channels must remain constant throughout the flow net. For the two examples of embankments with foundations of differing permeabilities, figures 4-13 and 4-14, the flow is more or less parallel to the interface, and the more permeable zone will dominate flow location and quantity. Because of this, the flow net can be started by assuming all flow goes through the most permeable zone. Once this flow net is drawn, it is extended into the lower permeability zone and refined to meet the general flow net criteria of paragraph 4.6. Transferring a line of seepage across the interface of soils of differing permeability, such as in a zoned dam, is more involved than transferring of flow lines and equipotential lines and will be described in Chapter 6.

4-9. Determination of Seepage Quantities, Escape Gradients, Seepage Forces, and Uplift Pressures. A flow net is a picture of seepage conditions under given geometry and boundary conditions. It explains how pressures are distributed and where flow is being directed. Coupled with the knowledge of head imposed on and the permeability of the porous media, the flow net can supply important information about stability and flow quantity in two-dimensional idealization of the real situation.

a. Seepage Quantities. Each of the complete flow channels passes an equal volume of water per unit of time, while partial channels carry a proportional flow. Each of the complete potential drops between equipotential lines is an equal portion of the total head, h , applied across the flow net with partial drops having a proportionally smaller part. The number of flow channels, including any partial channel, is given the symbol N_f while the number of equipotential drops, including any partial drops, is given the symbol N_d . The ratio of N_f/N_d is called the shape factor, $\$$, which is a characteristic of the given geometry and boundary conditions and permeability ratios (k_1/k_2, k_v/k_h). Quantity of flow per unit length through the porous media can be determined by using Darcy's law, q = kiA and the shape factor. Total flow is the sum of the flows through each flow channel, i.e., $q = \Sigma\Delta q = N_f\Delta q$ where q is the total flow and Δq is the flow through each complete flow channel. In figure 4-10, $q = N_f\Delta q = 6.65\Delta q$. Since Δh is the head loss between each equipotential line ($h = N_d\Delta h$) and $\Delta \ell$ is the dimension of a flow net square:

$$i = \frac{\Delta h}{\Delta \ell}$$

and from the Darcy equation:

$$\Delta q = k \frac{\Delta h}{\Delta \ell} a$$

where a is the area of the rectangle perpendicular to the flow direction. If one side of the rectangle is one unit of length perpendicular of the plane of the flow net, and the other dimension is $\Delta \ell$, thus a = $\Delta \ell$ (1) . This leads to:

$$\Delta q = k \frac{\Delta h}{\Delta \ell} \Delta \ell \ (1) = k\Delta h \ (1)$$

$$= k \frac{h}{N_d} (1)$$

Then:

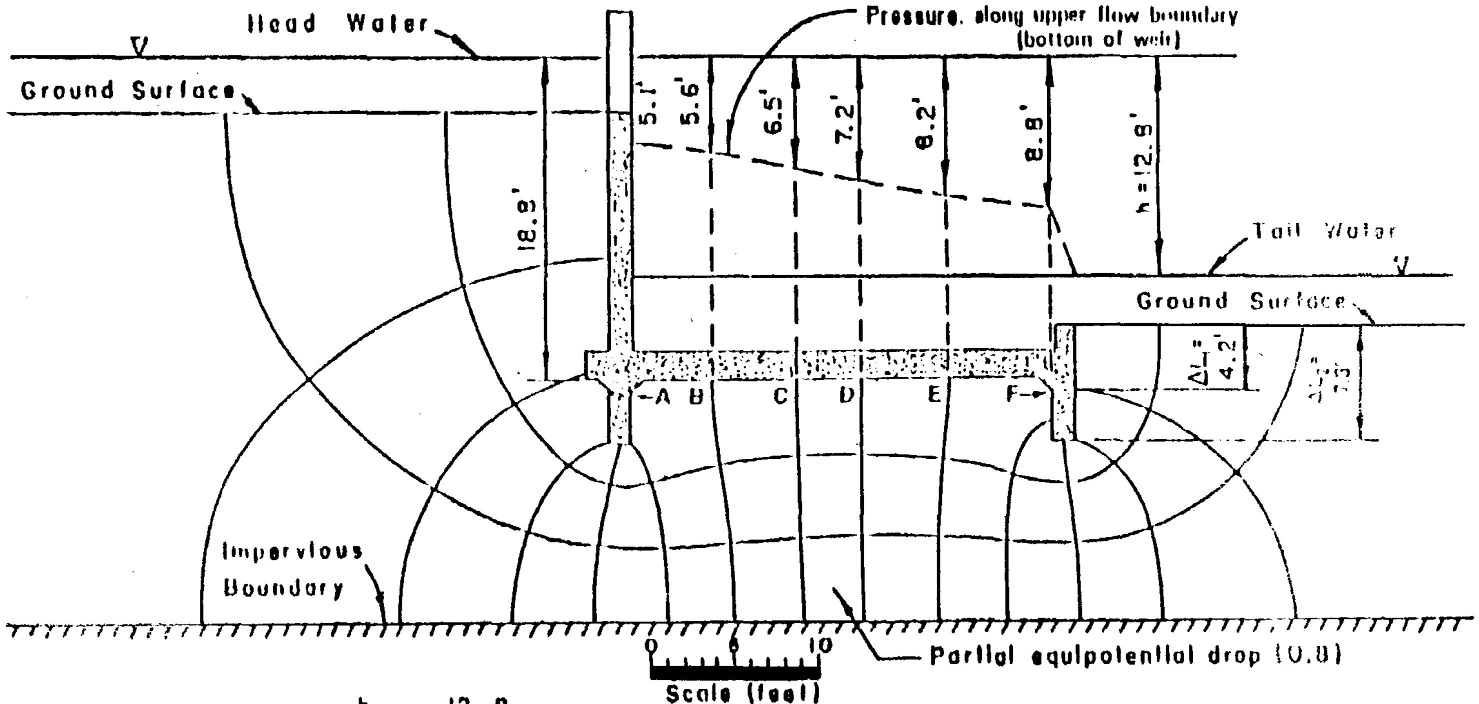
$$q = \Delta q N_f$$

$$q = k \frac{h}{N_d} N_f \ (1)$$

$$q = k\emptyset h \ (1) \text{ or } k\emptyset h$$

which gives the quantity of seepage flow for each unit of thickness of porous media perpendicular to the plane of the flow net. Figure 4-10(b) gives an example of this calculation for anisotropic seepage conditions in a dam foundation. The permeability, k' , used for anisotropic conditions, $k' = \sqrt{k_v k_h}$, is derived by Casagrande (1937).

b. Escape and Critical Gradients. The escape or exit gradient, i_e , is the rate of dissipation of head per unit of length in the area where seepage is exiting the porous media. For confined flow, the area of concern is usually along the uppermost flow line near the flow exit, e.g., at the downstream edge of a concrete or other impermeable structure, figure 4-15. Escape gradients for flow through embankments may also be studied by choosing squares from the area of interest in the flow net (usually at or near the exit face and downstream toe) and calculating gradients. If the gradient is too great where seepage is exiting, soil particles may be removed from this area. This phenomenon, called flotation, can cause piping (the removal of soil particles by moving water) which can lead to undermining and loss of the structure. The gradient at which flotation of particles begins is termed the critical gradient, i_{cr} . Critical gradient is determined by the in-place



$N_d = 13.8;\ \Delta h = \dfrac{h}{N_d} = \dfrac{12.8}{13.8} = 0.93'$

Point designation	No. of drops to point	Head loss = ($\Delta h \times$ No. drops) (ft.)	Uplift pressure (18.8' - hd. loss) γ_w (p.s.f.)
A	5.5	5.1	855
B	6	5.6	824
C	7	6.5	768
D	7.8	7.2	724
E	8.8	8.2	661
F	9.5	8.8	624

Escape gradient at toewall and transverse sill:

$$i_{e_1} = \frac{\Delta h}{\Delta L_1} = \frac{0.93}{4.2} = 0.22$$

$$i_{e_2} = \frac{2\Delta h}{\Delta L_2} = \frac{1.86}{7.3} = 0.25$$

Figure 4-15. Base uplift and escape gradient for concrete drop spillway (from U. S. Department of Agriculture[123]) unit weight of the soil and is the gradient at which upward drag forces on the soil particles equal the submerged weight of the soil particles, figure 4-16. The critical gradient is dependent on the specific gravity and density of the soil particles and can be defined in terms of specific gravity of solids, G_s , void ratio, e , and porosity, n :

$$i_{cr} = \frac{\gamma'_m}{\gamma_w} \quad \frac{G_s(1 - n)\gamma_w + n\gamma_w - \gamma_w}{\gamma_w}$$

$$= G_s(1 - n) + n - 1 = G_s\ (1 - n) - (1 - n)$$

$$i_{cr} = (G_s - 1)(1 - n)$$

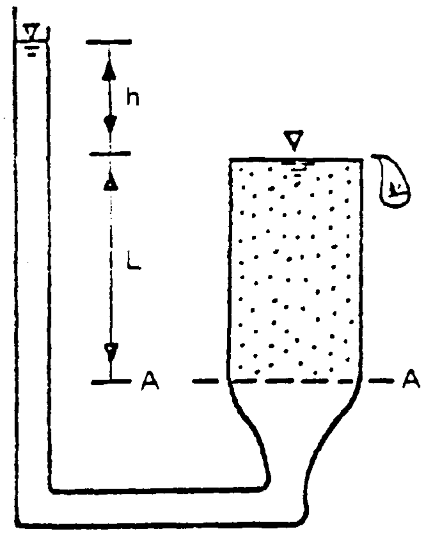
or, since $e = \frac{n}{1-n}$ and $n = \frac{e}{1 + e}$

$$i_{cr} = (G_s - 1)\left(\frac{n}{e}\right) = (G_s - 1)\ \frac{\frac{e}{1 + e}}{e}$$

$$i_{cr} = \frac{G_s - 1}{1 + e}$$

If typical values of G_s , e , and n for sand are used in the above equations, i_{cr} will be approximately 1. Investigators have recommended ranges for factor of safety for escape gradient, $FS_G = \frac{i_{cr}}{i_e}$ from 1.5 and 15, depending on knowledge of soil and possible seepage conditions. Generally, factors of safety in the range of 4-5 (Harr 1962, 1977) or 2.5-3 (Cedergren 1977) have been proposed.

c. Heave. In some cases, movement of soil at the downstream seepage exit may not occur as flotation followed by particle-by-particle movement. A mass of soil may be lifted initially, followed by piping. This phenomenon is called heave and occurs when the upward seepage force due to differential head equals the overlying buoyant weight of soil. Heave occurs under conditions of critical hydraulic gradient. For field conditions, the point at which minimum differential head offsets the overlying buoyant weight must be determined by judgment and calculations. Terzaghi and Peck (1967) have evaluated the factor of safety with respect to heave for a row of sheet piles. Resistance to heave



A = AREA OF SOIL SAMPLE
L = LENGTH OF SOIL SAMPLE
γ_m = UNIT WEIGHT OF SATURATED SOIL
γ_w = UNIT WEIGHT OF WATER
$\gamma_m' = \gamma_m - \gamma_w$ = BOUYANT UNIT WEIGHT OF SOIL
h = DIFFERENTIAL HEAD
i_{cr} = CRITICAL GRADIENT

<u>FORCES ACTING ON PLANE A-A</u>

FORCES DOWN, $F_d = \gamma_m L A$ FORCES UP, $F_u = (h+L)\gamma_w A$

WHEN $F_d = F_u$ FLOTATION TAKES PLACE WHICH IS THE CONDITION FOR CRITICAL GRADIENT, i_{cr}

$$\gamma_m L A = (h+L)\gamma_w A$$

$$\frac{\gamma_m}{\gamma_w} = \frac{h+L}{L} = \frac{h}{L} + 1$$

$$\frac{h}{L} = \frac{\gamma_m}{\gamma_w} - 1 = \frac{\gamma_m - \gamma_w}{\gamma_w} = \frac{\gamma_m'}{\gamma_w} = i_{cr}$$

THUS WHEN $i = \frac{h}{L} = \frac{\gamma_m'}{\gamma_w}$ THE GRADIENT IS CRITICAL (i_{cr})

Figure 4-16. Definition of critical gradient (prepared by WES)

may be developed by placing very pervious material on the exit face, which will allow free passage of water but add weight to the exit face and thus add downward force. This very pervious material must meet filter criteria to prevent loss of the underlying soil through the weighting material.

d. <u>Seepage Forces</u>. Forces imposed on soil particles by the drag of water flowing between them must be considered when analyzing the stability of slopes, embankments, and structures subject to pressures from earth masses. These forces are called seepage forces. The magnitude of this force on a mass of soil is determined by the difference in piezometric head on each side of the soil mass, the weight of water, and the area perpendicular to flow. The seepage force acts in the same direction as flow, i.e. along flow lines. Consider the seepage force on plane A-A in figure 4-16. Since flow is vertically upward, the direction of seepage force is up, the difference in piezometric head is h , and the area perpendicular to flow is A . The seepage force, F_s , is the part of the upward forces due to differential head, h , or:

$$F_s = h\gamma_w A$$

In terms of gradient and unit volume;

$$F_s = \frac{h}{L}\gamma_w \ AL = i\ \gamma_w\ V$$

and using f_s as seepage force per unit volume:

$$f_s = \frac{F_s}{V} = i\gamma_w$$

Two methods of applying this force to use in stability analysis are described by Cedergren (1977) and termed the gradient method and boundary pressure method. EM 1110-2-1902 gives examples of embankment stability analyses considering seepage forces. Additionally, the effect of buoyant forces on soil mass stability must also be considered. The upward or buoyant force, F_b , causing reduction in effective stress on plane A-A, figure 4-15, is the remainder of the upward forces on plane A-A:

$$f_b = L\gamma_w A$$

e. Uplift Pressures. When seepage occurs beneath concrete or other impermeable structures or strata, the underside of this impermeable barrier is subject to a force which tends to lift the structure upward. The determination of this pressure or force is important in analyzing the stability of the structure. An example of the analysis is given in figure 4-15. Summing of the uplift pressures over the bottom area of the spillway will give the total uplift force on the structure for a stability analysis. Harr's text (1962) provides methods other than flow net construction to determine uplift pressures.